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Procedure for Cyclic  
Structural Analysis**

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**Development of a Simplified  
Procedure for Cyclic  
Structural Analysis**

Albert Kaufman

*Lewis Research Center  
Cleveland, Ohio*



National Aeronautics  
and Space Administration

Scientific and Technical  
Information Branch

## Summary

Nonlinear, finite-element computer programs are too costly to use in the early design stages of hot section components for aircraft gas turbine engines. To improve the durability of these components, it is necessary to develop simpler and more economical methods for representing the structural response of materials under cyclic loading. This study was conducted to develop a computer program to perform a simplified nonlinear structural analysis using only a calculated or constructed elastic solution as input data.

The simplified method was based on the assumption that the inelastic regions in the structure are constrained against strain redistribution by the surrounding elastic material; therefore, the total strain history can be defined by an elastic analysis. A computer program (ANSYMP) was created to predict the stress-strain history at the critical fatigue location of a thermomechanically cycled structure from elastic input data. Appropriate material stress-strain and creep properties and a plasticity hardening model were incorporated in the program. Effective stresses and plastic strains are approximated by an iterative and incremental solution procedure. Creep effects can be obtained on the basis of stress relaxation at constant strain, cumulative creep at constant stress, or a combination of stress relaxation and creep accumulation.

Variations of three problems were examined analytically to verify the accuracy of the simplified method. Verification was made through comparison with a three-dimensional, nonlinear, finite-element analysis (MARC). These problems were (1) a uniaxial specimen subjected to strain cycling under isothermal conditions; (2) a benchmark notch specimen subjected to load cycling; and (3) a prismatic wedge specimen subjected to thermal cycling. Cyclic stress-strain and creep properties for Inconel 718 alloy and a kinematic hardening model were used for the uniaxial and benchmark notch specimen problems. Cyclic stress-strain and creep properties for IN 100 alloy and a kinematic hardening model were assumed for the wedge specimen problem.

Elastic and nonlinear finite-element analyses, with and without dwell times, were performed for all three cases using the MARC computer program. The elastic solutions for the critical locations were used as input data for the simplified analysis computer program. Comparisons were made of the stress-strain histories at the critical locations as calculated from the simplified and nonlinear finite-element analyses.

The comparisons demonstrated that the simplified method can duplicate the cyclic stress-strain hysteresis loops from MARC elastic-plastic analyses to a high degree of accuracy. Agreement between ANSYMP and MARC analyses involving creep dwell times were generally good. However, the ANSYMP creep computations were very sensitive to variations in the

calculated effective stresses. Mean stresses calculated from the simplified method were in good agreement with the MARC results. In a typical problem, ANSYMP used less than 1 percent of the central processor unit (CPU) time required by MARC to compute the inelastic solution.

## Introduction

The drive toward better performance and fuel economy for aircraft gas turbine engines has resulted in higher turbine inlet temperatures, pressure ratios, and rotor speeds. These more severe operating conditions have subjected the hot section components to thermomechanical load cycles that induce significant inelastic strains and eventual fatigue cracking. It has become increasingly difficult to design reliable components to meet both the engine life and performance requirements. Improvements in the durability of these components depend on accurate structural analysis and life prediction. Life prediction methods have been under development by the NASA Lewis Research Center and other organizations (refs. 1 to 4). Applying these methods requires knowledge of the temperature—stress-strain history at the critical crack initiation location of the structure.

The primary structural parameters of interest for life prediction purposes are the total strain range and the mean cyclic stress. For most practical cases, the critical location and the total strain range can be satisfactorily obtained from an elastic analysis as demonstrated in references 3 to 9. However, in cases involving purely mechanical load cycling, creep, or large plastic strains, an elastic analysis may not be adequate to determine the total strain range. Mean stresses for hot section components, as well as multiaxial and thermomechanical fatigue specimens, must be calculated from some type of nonlinear analysis. The accuracy of the solutions is largely dependent on the adequacy of the creep-plasticity models and the material properties used for the analysis.

Nonlinear finite-element analysis is being used increasingly for calculating inelastic structural response. However, nonlinear methods are not feasible for use as a component design tool because of the high computing costs associated with the iterative and incremental nature of the inelastic solutions. Computing costs are further increased by the presence of high thermal gradients and geometrical irregularities, such as cooling holes, which necessitate three-dimensional analyses. Three-dimensional, nonlinear finite-element analyses are prohibitively time consuming and expensive to conduct in the early design stages for combustor and turbine structures.

To improve the design of hot section components, it is necessary to develop simpler and more economical

methods for representing structural behavior under cyclic loading. Development of life prediction methods would also benefit from a simplified analysis method for determining the structural behavior of multiaxial and thermomechanical fatigue specimens.

A study was conducted to develop a fully automated simplified analytical procedure for estimating the stress-strain history of a thermomechanically loaded structure subject to cyclic inelasticity. The initial development of the simplified procedure, which was limited to consideration of plasticity effects, is discussed in reference 10. This simplified procedure was implemented in a computer program (ANSYMP).

The ANSYMP program was created to predict the stress-strain response at the critical location of a thermomechanically cycled structure from a calculated elastic solution or one constructed from strain measurements at the critical location. An incremental and iterative procedure estimates the plastic strains from the material stress-strain properties and a plasticity hardening model. Analytical predictions from the simplified method were compared with nonlinear finite-element solutions from the MARC computer program (ref. 11) for a number of cases. These involved uniaxial and multiaxial stress states, isothermal and nonisothermal conditions, and various materials and plasticity hardening models. The problems included an Inconel 718 benchmark notch specimen that was load cycled in an experiment to verify structural analysis methodologies (ref. 12). Nonlinear analyses using the MARC program were performed for the benchmark problem in the study reported in reference 5. A kinematic hardening model was found to give excellent agreement with the experimental results for this specimen under continuous mechanical load cycling. Another case for which the simplified method was evaluated was a double-edge wedge specimen which had been thermally cycled in fluidized beds (ref. 13). MARC nonlinear analysis results for the test conditions are reported in reference 6.

In the present study, the simplified method was further developed to consider creep and stress relaxation effects. The computational algorithms in the original ANSYMP program, involving reversed plasticity under nonisothermal conditions, were also improved. Creep options were incorporated in the program on the basis of stress relaxation at constant strain, creep at constant stress, or a combination of stress relaxation and creep accumulation.

Variations of three problems were exercised with the ANSYMP elastic-plastic-creep program. These included a strain-controlled uniaxial problem using cyclic stress-strain and creep properties for Inconel 718 alloy, as well as the benchmark notch and wedge specimen problems with and without creep dwell times. In this study, cyclic stress-strain and creep properties for IN 100 alloy and a kinematic hardening model were used in the analyses for

the wedge specimen problem. Verification of the ANSYMP program was made on the basis of how well it was able to duplicate the stress-strain hysteresis loops from MARC elastic-plastic-creep analyses of these problems.

## Symbols

The symbols are designated in Fortran coding, as much as possible, to facilitate following a computer program flow chart (fig. 2).

A,B,C	temperature-dependent constants in creep power law, eq. (5)
DELY	yield stress shift due to load reversal
EC	creep strain
EMOD	modulus of elasticity
EP	plastic strain
EP'	maximum plastic strain in cycle (fig. 1)
ETOT	total strain
K,n	temperature-dependent constants in stress-strain exponential eq. (1)
SIGMA	stress
SIGMA'	maximum stress in cycle (fig. 1)
SIGY	current yield stress
SIGY1	initial yield stress
SLOPE	kinematic work hardening slope (fig. 1)
$\nu$	Poisson's ratio

## Analytical Procedure

A simplified inelastic procedure was developed for calculating the stress-strain history at the critical fatigue location of a structure subjected to cyclic thermomechanical loading. The fundamental assumption in this procedure is that the inelastic region is local and is constrained from redistribution by the surrounding elastic material. It follows from this assumption that the total strain history at the critical location can be defined by an elastic solution. Justification for the assumption of elastic constraint of local inelasticity can be found in references 3 to 9, where structural analyses of combustor liners, air-cooled turbine blades, and wedge fatigue specimens have shown that the total strain ranges from elastic and nonlinear solutions are in close agreement. A corollary to this assumption is that the elastic loading and unloading segments of the effective stress-equivalent total strain hysteresis loops constructed from an elastic-plastic analysis will be parallel to the elastic hysteresis loop. This is documented by comparing the nonlinear and elastic hysteresis loops in references 5 and 6.

The basic problem in developing the simplified analytical procedure was to characterize the yield surface in terms of the total strain obtained from an elastic analysis or strain measurements. Classical plasticity theory characterizes the yield surface by a yield condition to describe yielding under multiaxial stress states and by a hardening model to establish the location of the yield surface during cycling. The simplified procedure was set up to accommodate itself to any yield criterion or hardening model. The only requirements are that the elastic input data be consistent with the yield criterion and that the appropriate material properties be used in conjunction with the hardening model.

In this study, all the analyses were performed with a kinematic hardening model. A representation of a cyclic stress-strain curve by a bilinear kinematic hardening model is illustrated in figure 1. The loci of the tips of the cyclic curves are described by the exponential equation

$$\text{SIGMA} = K(\text{EP})^n \quad (1)$$

The work-hardening slope for the kinematic hardening model was determined from energy considerations to give the same strain energy (indicated by the enclosed area in fig. 1) as the actual stress-strain curve. This work hardening slope is defined by

$$\text{SLOPE} = (\text{SIGMA}'/\text{EP}') (2n/(1+n)) \quad (2)$$

and the initial yield point SIGY1 by

$$\text{SIGY1} = \text{SIGMA}' - \text{SLOPE}(\text{EP}') \quad (3)$$

The yield stress shift DELY due to load reversal under kinematic hardening is

$$\text{DELY} = 2(\text{SIGY} - \text{SLOPE}(\text{EP})) = 2(\text{SIGY1}) \quad (4)$$

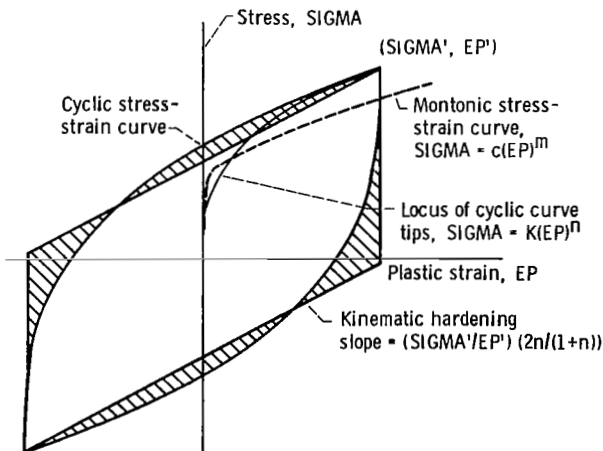


Figure 1. — Representation of stress-strain curves.

Creep characteristics of the material were incorporated in the program in the form

$$\text{EC} = (\text{SIGMA}/A)^{BtC} \quad (5)$$

A strain-hardening law (ref. 14) was used to accumulate creep strain under changing stress. Any one of three creep options can be selected: (1) stress relaxation at constant strain, (2) cumulative creep at constant stress, and (3) a combination of (1) and (2).

Most nonlinear computer programs use the von Mises yield criterion and incremental plasticity theory. Implicit in the von Mises yield criterion is the conversion of the total strain from a uniaxial stress-strain curve to a modified equivalent total strain, as discussed in reference 14. The modified elastic equivalent total strain corresponds to the uniaxial total elastic strain multiplied by  $2(1+\nu)/3$ . This relationship must be taken into account for multiaxial problems in applying strain results from elastic finite-element programs or strain measurements as input for the simplified inelastic analysis. Both elastic and nonlinear finite-element analyses for this study were conducted with the MARC computer program. The elastic solutions computed from MARC for input into the simplified analysis method were automatically obtained in terms of von Mises effective stresses and modified equivalent total strains.

The elastic input data are subdivided into a sufficient number of increments to define the stress-strain cycle. Dwell times are specified for increments which require creep analysis. The increments are analyzed sequentially to obtain the cumulative plastic and creep strains and to track the yield surface. An iterative procedure is used to calculate the yield stresses for increments undergoing plastic straining. First, an estimated plastic strain is assumed for calculating an initial yield stress from the stress-strain properties and the simulated hardening model. Second, a new plastic strain is calculated as

$$\text{EP} = \text{ETOT} - \text{EC} - \text{SIGY}/\text{EMOD} \quad (6)$$

The yield stress is then recalculated using the new plastic strain. This iterative procedure is repeated until the new and previous plastic strains agree within a 1-percent tolerance.

A FORTRAN IV computer program (ANSYMP) was created to automatically implement the simplified analytical procedure. The program consists of the main executive routine ANSYMP and the four subroutines ELAS, YIELD, CREEP, and SHIFT. The incremental elastic data and temperatures are read into subroutine ELAS. Material stress-strain properties as functions of temperature and a simulated hardening model are incorporated in subroutine YIELD and the creep characteristics are incorporated in subroutine CREEP. Subroutine SHIFT is required to update the temperature

effects on the yield stress shift; this was omitted in the original program described in reference 10 and constitutes an error for problems involving plastic reversal due to thermal loading. SHIFT also serves the function of deciding the future direction of the yield surface under nonisothermal conditions by determining the relation of future to past thermal loading.

The ANSYMP program is available from the Computer Software Management Information Center (COSMIC), University of Georgia, Athens, Georgia 30602, under LEW 14011. Figure 2 is a flow chart of the program. Sample input and output data are shown in tables I and II, respectively. These data are given in English units to be consistent with the material property coding in the ANSYMP program. At the first increment where yielding occurs, as in increment 4 in table II, the solution is shown for the initial yield point in order to fully define the stress-strain hysteresis loop.

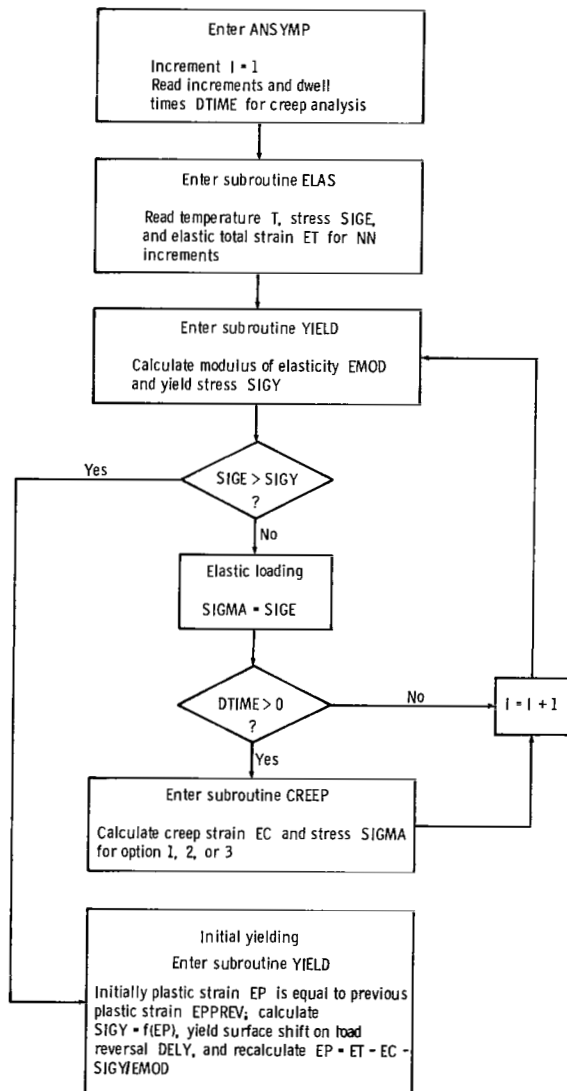


Figure 2. - Program flow chart.

The calculational scheme initially follows the effective stress—equivalent strain input data from subroutine ELAS until the occurrence of initial yielding. The stress-strain solution then proceeds along the yield surface as determined from the stress-strain properties in subroutine YIELD. At each increment during yielding the stress shift (difference between new yield stress and stress predicted from elastic analysis) from the original input data is calculated. Elastic load reversal is signaled when the input stress is less than the yield stress from the previous increment. During elastic unloading, the stresses are translated from the original elastic analysis solution by the amount of the calculated stress shift. Reverse yielding occurs when the stress reaches the reverse yield surface as determined from the hardening model incorporated in subroutine YIELD. Again, the solution follows the yield surface until another load reversal is indicated when the stress based on the shifted elastic solution is less than the yield stress. The elastic response during load reversal is obtained by translating the original elastic solution according to the new stress shift calculated during reversed yielding. The stress-strain response for

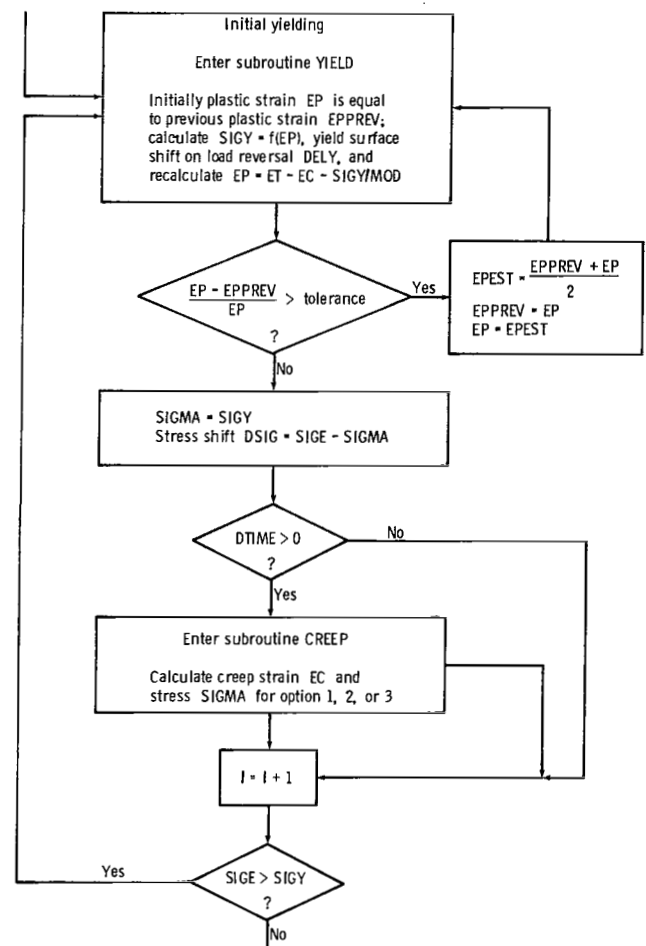


Figure 2. - Continued.

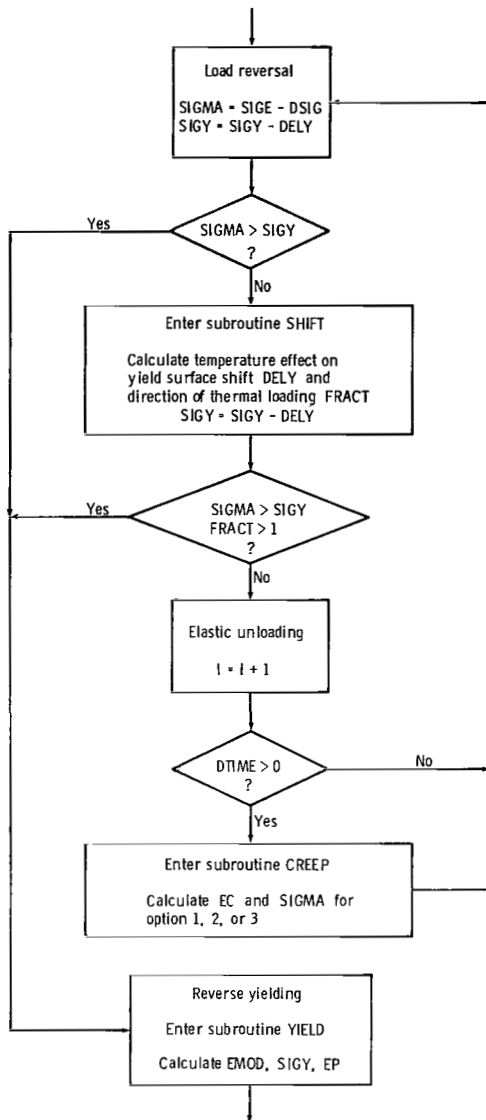


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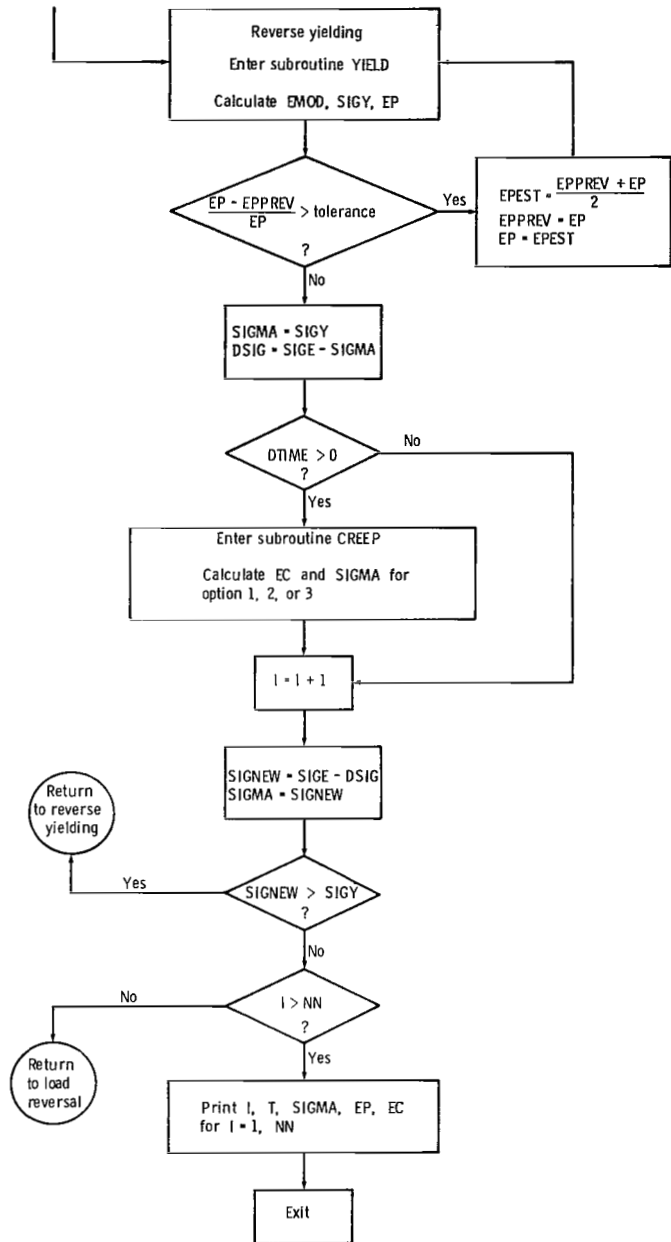


Figure 2. — Concluded.

subsequent cycles is computed by repeating this procedure of identifying load reversals, tracking reverse yield surfaces, and translating the original elastic solution during elastic loading and unloading. Creep computations are performed for increments involving dwell times using the creep equation and strain hardening rule incorporated in subroutine CREEP. Depending on the nature of the problem, the creep effects are determined on the basis of one of the three options provided in the subroutine.

The computer program was verified by conducting simplified analyses for a series of three problems and comparing the results to those from MARC nonlinear analyses. The first of these problems was a uniaxial specimen subjected to strain cycling under isothermal

conditions. Variations of this problem were run with no creep dwell times and with dwell times at minimum and intermediate total strain levels. A kinematic hardening model was used with cyclic stress-strain and creep properties for Inconel 718 alloy obtained from reference 12. Nonlinear and elastic MARC analyses of this problem were performed using a single 20-node, three-dimensional element. The MARC solutions for the uniaxial problem were computed for the centroid of the single solid-element model. The second problem considered was a mechanically load-cycled benchmark notch specimen shown in figure 3. This specimen was

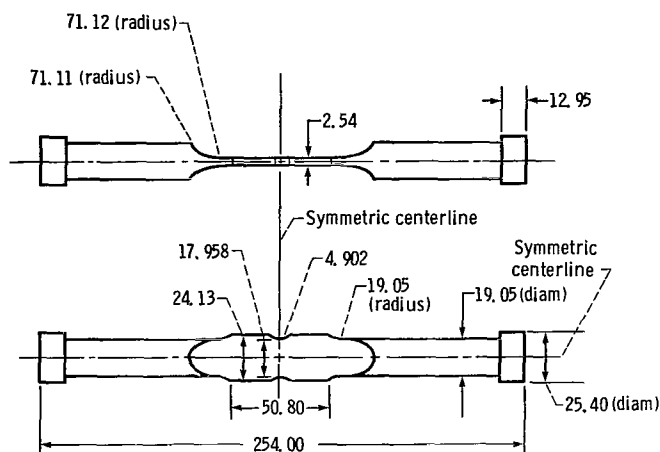


Figure 3. – Benchmark notch specimen ( $K_t = 1.9$ ). (All dimensions in mm.)

tested under isothermal conditions as part of a program to provide controlled strain data for constitutive model verification (ref. 12). A MARC analysis of this problem using kinematic hardening demonstrated excellent agreement with experimental data in reference 5. A number of variations of this problem were run with both the MARC and ANSYMP programs. These variations included dwell times at maximum, minimum, and intermediate total strains and dwell times at increments where tensile yielding occurred. The simplified analysis of the benchmark notch problem used the kinematic hardening model and cyclic stress-strain and data for Inconel 718 alloy given in reference 12. The third problem was an IN 100 double-edge wedge specimen that was thermally cycled in the fluidized bed facility discussed in reference 13. This problem provides a nonisothermal case for evaluating the computer program. Both the MARC and ANSYMP analyses used the kinematic hardening model and the IN 100 cyclic stress-strain and creep properties reported in reference 5. The geometry of the double-edge wedge specimen is illustrated in figure 4. The wedge problem was analyzed without and with dwell times at maximum and minimum total strain levels. The MARC solutions shown for the benchmark notch and wedge specimens were computed at the closest Gaussian integration point to the critical crack initiation location.

The material properties and kinematic hardening models were coded into subroutines YIELD, SHIFT, and CREEP. The sample input in table I and the output in table II are for the wedge specimen problem. The parameters NN, NCRP, ICRP, and ICREEP in table I refer, respectively, to the number of increments of elastic input data, number of increments with dwell times, number of subincrements the dwell times are to be subdivided into for creep calculations, and the selected creep option. KKK is a pointer that refers to the type of

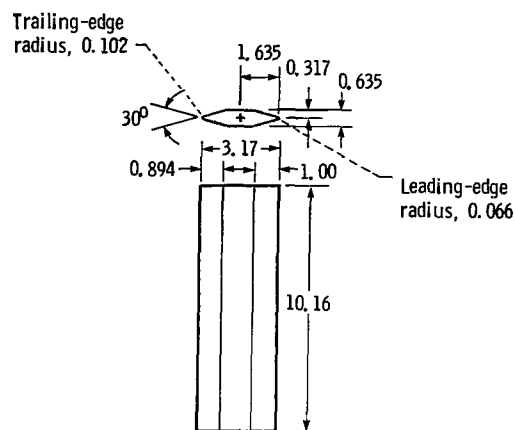


Figure 4. – Double-edge wedge. (All linear dimensions in cm.)

problem to be solved and the set of material properties to be used in the analysis. The temperature, stress, and total strain for the elastic solution are then listed for each increment. The elastic input data were repeated a second time to conduct the simplified analyses for two cycles for all the problems considered in this study. Finally, the dwell times are specified for those increments where creep calculations are to be performed. The sample output for this problem shown in table II includes an echo of the NN, NCRP, ICRP, ICREEP, and KKK parameters and the increment dwell times. For each increment, the temperature, stress, and the total, plastic, and creep strains are listed. It will be noted that the output in table II includes more increments than were shown for the input in table I; this is because the stress-strain solution is printed for the beginning and end of each dwell increment.

## Discussion of Analytical Results

The results of the simplified elastic-plastic-creep analyses of the uniaxial, benchmark notch, and wedge specimen cases are discussed herein. Comparisons are made with MARC inelastic solutions. The stress-strain cycles used for comparison purposes are in terms of effective stresses and equivalent total strains based on the von Mises yield criterion. The discussion is based on the critical location in the specimen where fatigue cracking would start.

### Uniaxial Problem

The uniaxial problem was used for the basic development of the simplified approach and computer program. Since the loading was strain-controlled, the maximum and minimum total strains were identical for the MARC elastic and nonlinear finite-element solutions. Also, the effect of creep dwell time at any increment



caused stress relaxation under constant total strain. In reference 10 excellent agreement was demonstrated between ANSYMP and MARC elastic-plastic analyses for uniaxial isothermal cases involving initial tensile loading, initial compressive loading, and imposed strain ratchetting.

Four variations of the uniaxial problem were considered in this study. These were initial compressive loading without creep dwell times (fig. 5(a)), dwell time at maximum strain (fig. 5(b)), dwell time at minimum strain (fig. 5(c)), and dwell times at minimum and intermediate strains (fig. 5(d)). A constant temperature of 649° C was assumed during the strain cycling. Creep option (1)

(stress relaxation at constant strain) was used for all the creep computations.

A comparison of the stress-strain cycles obtained from the simplified and MARC elastic-plastic-creep analyses is shown in figure 5. Agreement between the ANSYMP and MARC nonlinear solutions is seen to be excellent for all the uniaxial cases.

### Benchmark Notch Problem

The benchmark notch test was conducted by mechanical load cycling at a constant temperature of 649° C. A mechanically loaded structure, especially

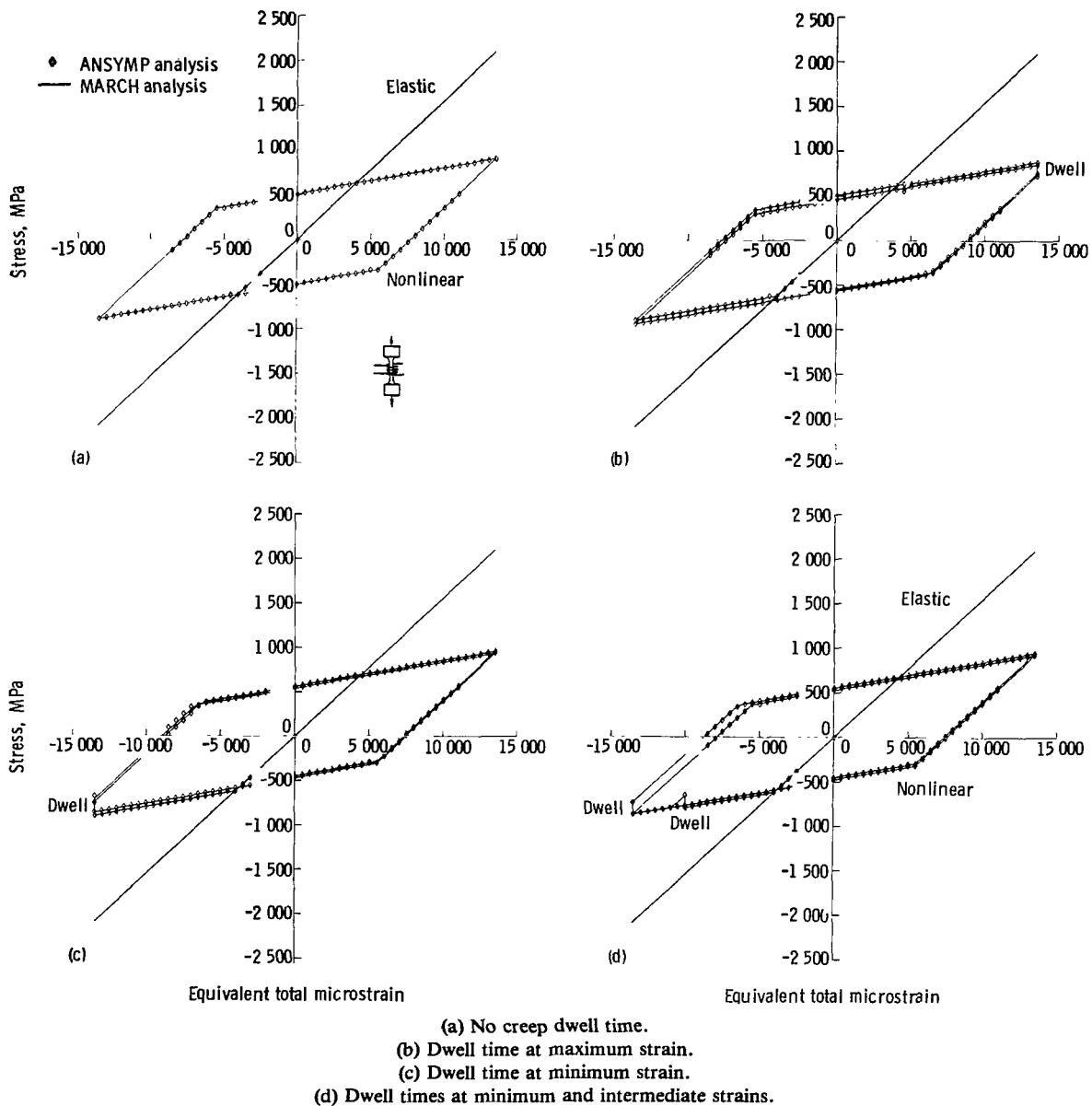


Figure 5. — Comparison of ANSYMP and MARC analysis results for uniaxial problem.

where the peak strain occurs at a stress raiser, is most likely to violate the basic assumption of the simplified approach that strain redistribution is prevented by containment of the local plastic region by the surrounding elastic material. It was shown in reference 10 that the total strain range from the MARC elastic-plastic analysis was 20 percent greater than that obtained from the MARC elastic analysis. This foreshortening of the elastic strain range caused the simplified procedure to truncate the stress-strain hysteresis loop. When the elastic solution was extended to be consistent with the measured

notch root strain, the agreement between the simplified and MARC elastic-plastic stress-strain hysteresis loops was excellent as demonstrated in figure 6(a). Both the ANSYMP and MARC elastic-plastic analyses gave stable stress-strain hysteresis loops for the second cycle. Further study is required to develop rules or guidelines for adjusting the elastic solution in this type of problem. The extended elastic strain range was used for all analyses of the benchmark notch problem in this study.

In figure 6 comparisons are shown of simplified and nonlinear finite-element analytical results for benchmark

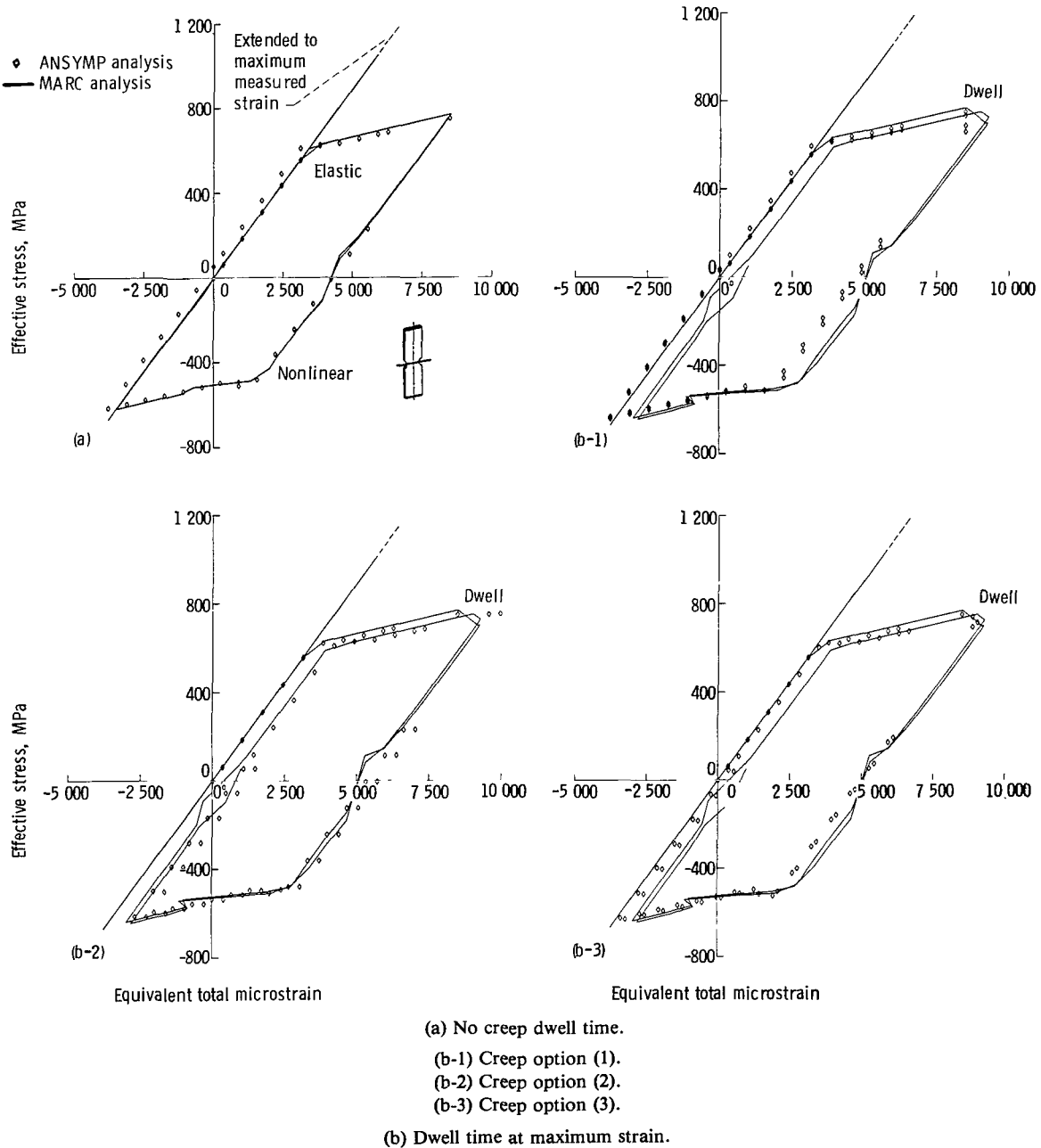
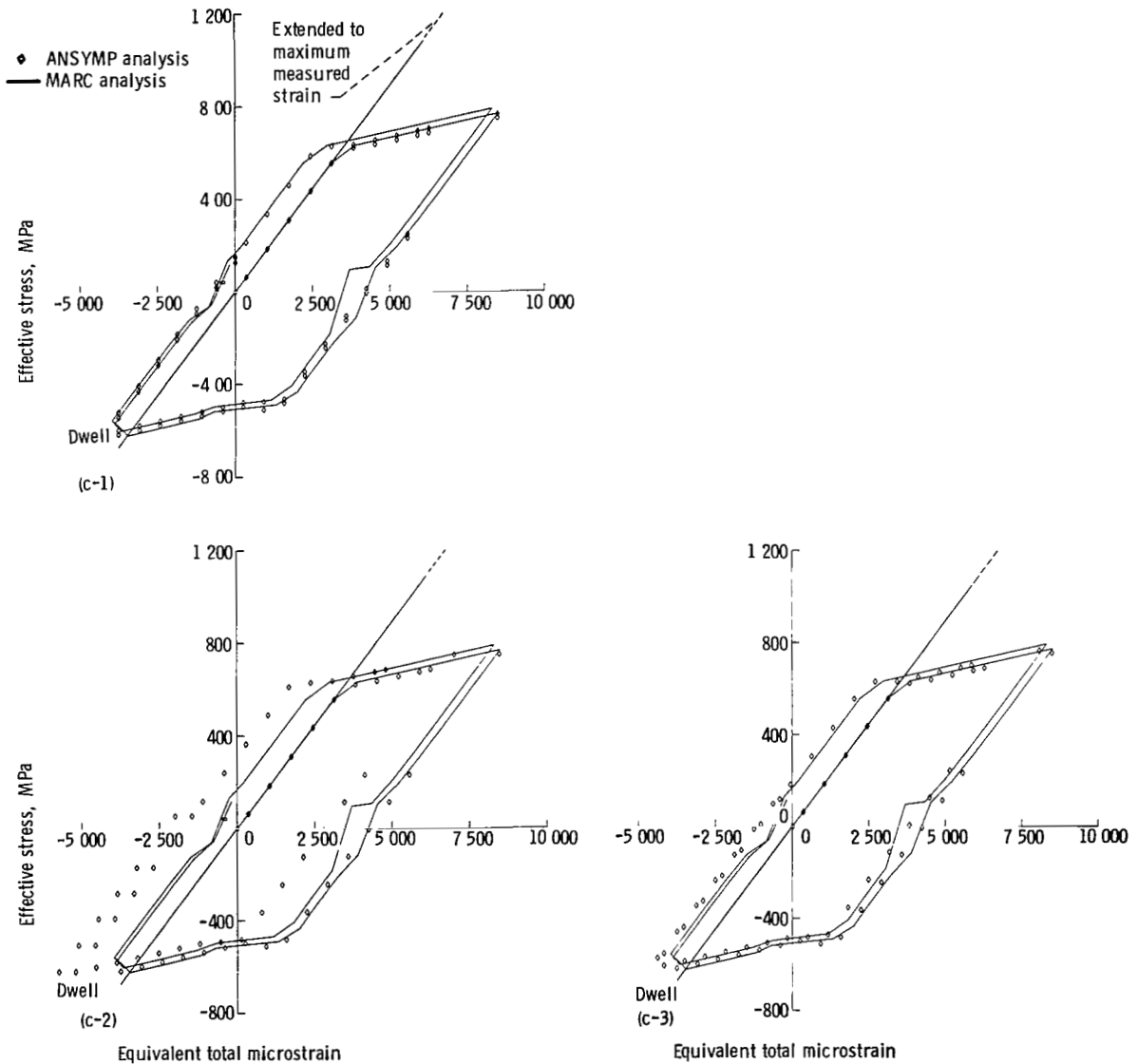


Figure 6. - Comparison of ANSYMP and MARC analysis results for benchmark problem.

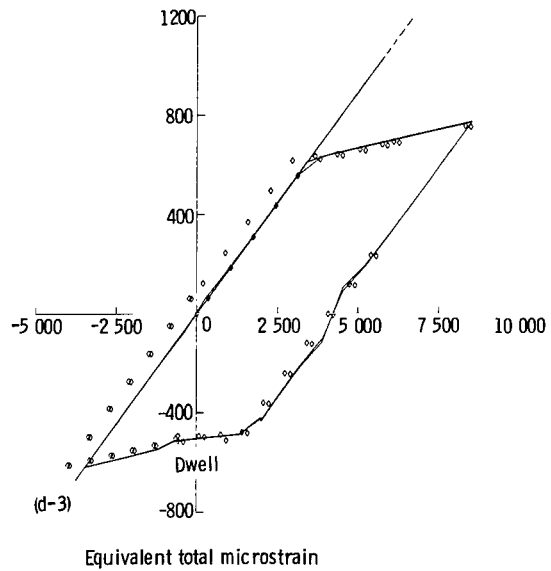
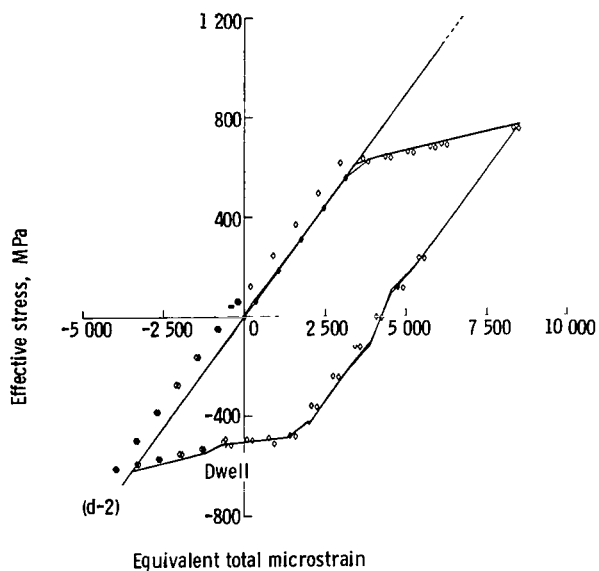
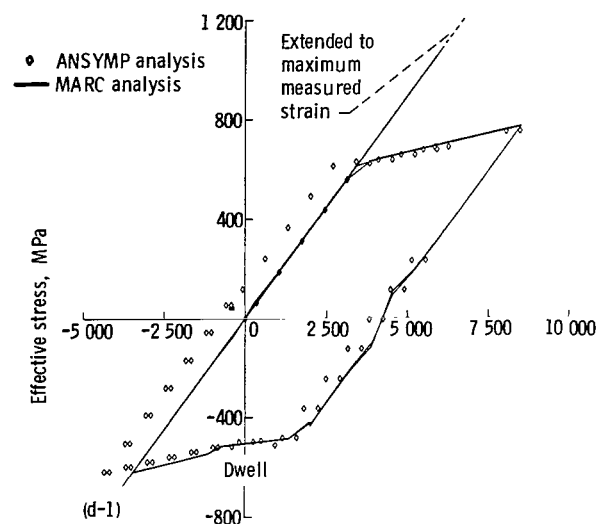
notch cases involving dwell times at maximum strain (fig. 6(b)), at minimum strain (fig. 6(c)), at intermediate strain in compressive yield (fig. 6(d)), and at all increments involving tensile yielding (fig. 6(e)). ANSYMP analyses were performed using all three creep options for each benchmark case. As demonstrated in figure 6, the creep analyses using option (3) (combined stress relaxation and creep accumulation) gave the most consistent agreement with the MARC nonlinear finite-element solutions. This would indicate that creep option (3) should be used in most cases other than strain-controlled problems.

In terms of cycle mean stresses, the simplified procedure gave results more compatible with MARC elastic-plastic analyses than were possible from an elastic solution. The mean stresses from the simplified and MARC elastic solutions were 68 and 223 megapascals, respectively, compared to 77 megapascals for the MARC elastic-plastic solution. The application of creep dwell times did not significantly alter the cycle mean stresses. The ANSYMP analyses of the benchmark notch problem used less than 1 percent of the central processor unit (CPU) time required by the MARC nonlinear analyses.



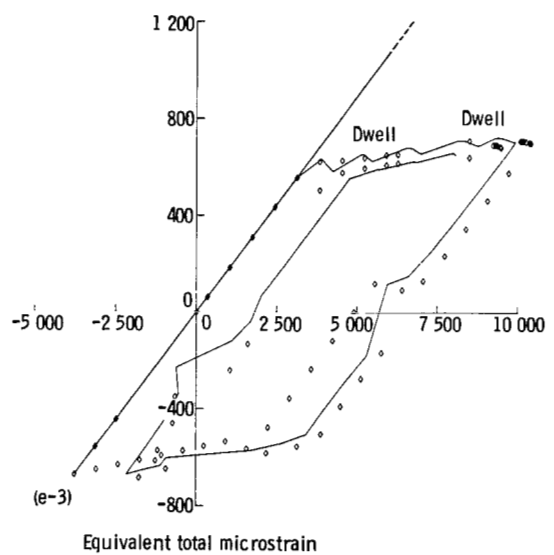
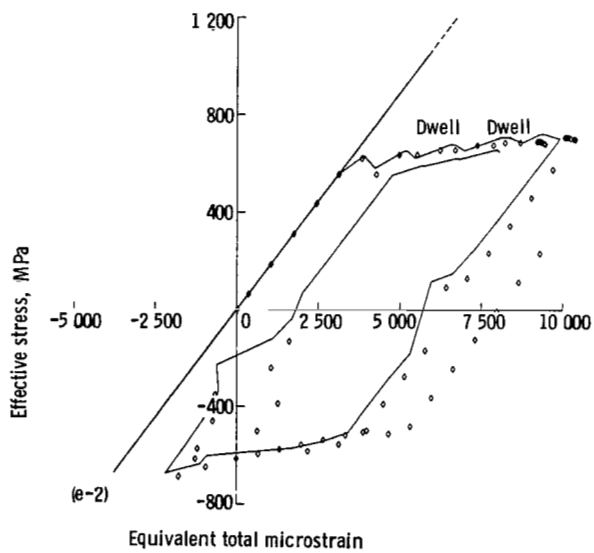
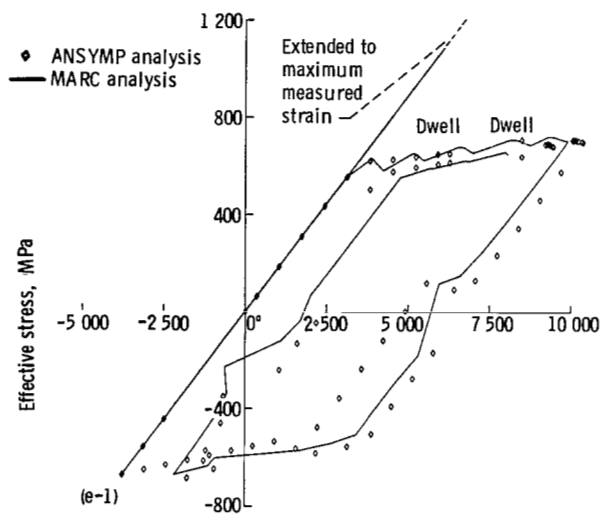
(c-1) Creep option (1).  
(c-2) Creep option (2).  
(c-3) Creep option (3).  
(c) Dwell time at minimum strain.

Figure 6. - Continued.



(d-1) Creep option (1).  
 (d-2) Creep option (2).  
 (d-3) Creep option (3).  
 (d) Dwell time at intermediate strain.

Figure 6. – Continued.



(e-1) Creep option (1).

(e-2) Creep option (2).

(e-3) Creep option (3).

(e) Dwell times at tensile yield strains.

Figure 6. – Concluded.

## Wedge Specimen Problem

The double-edge wedge specimen provided a nonisothermal case for evaluation of the simplified procedure and the operation of the ANSYMP program. Because of the incremental temperature changes, the elastic solution was no longer linear as it had been for the isothermal uniaxial and benchmark notch cases.

In figure 7(a), the stress-strain hysteresis loops calculated from the ANSYMP simplified procedure and MARC elastic-plastic analyses are compared for two thermal cycles without dwell times. Reasonably good

agreement is shown between the ANSYMP and MARC stress-strain hysteresis loops in figure 7(a). The mean stress for the second MARC stress-strain cycle was 55 megapascals. The simplified procedure predicted a mean stress of 20 megapascals compared to -201 megapascals for the elastic solution.

These analyses were repeated with dwell times imposed at the maximum strain level. As shown in figure 7(b), the predicted ANSYMP solutions for this case with all three creep options were not in good agreement with the MARC nonlinear stress-strain cycles. This was due to the

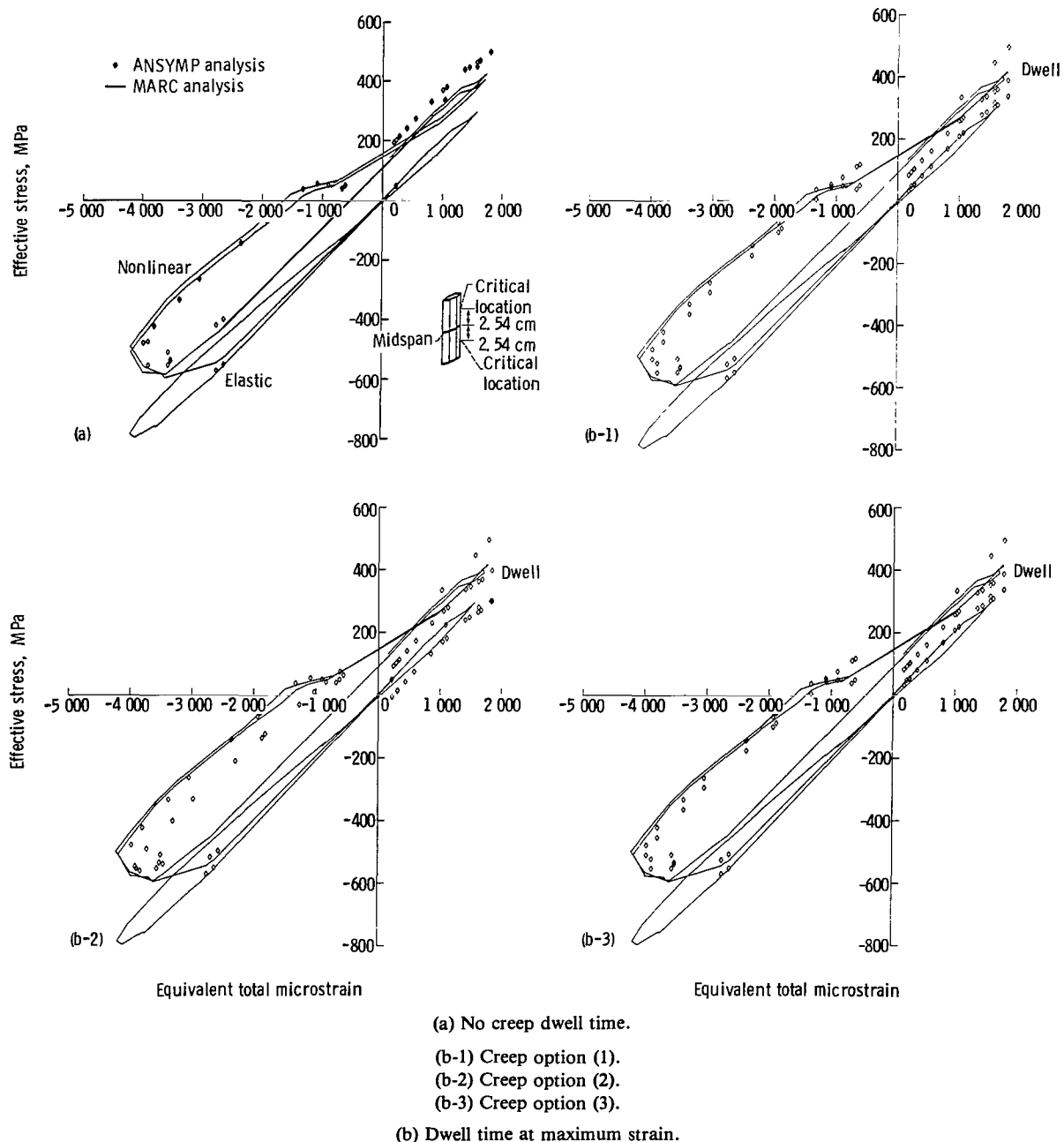
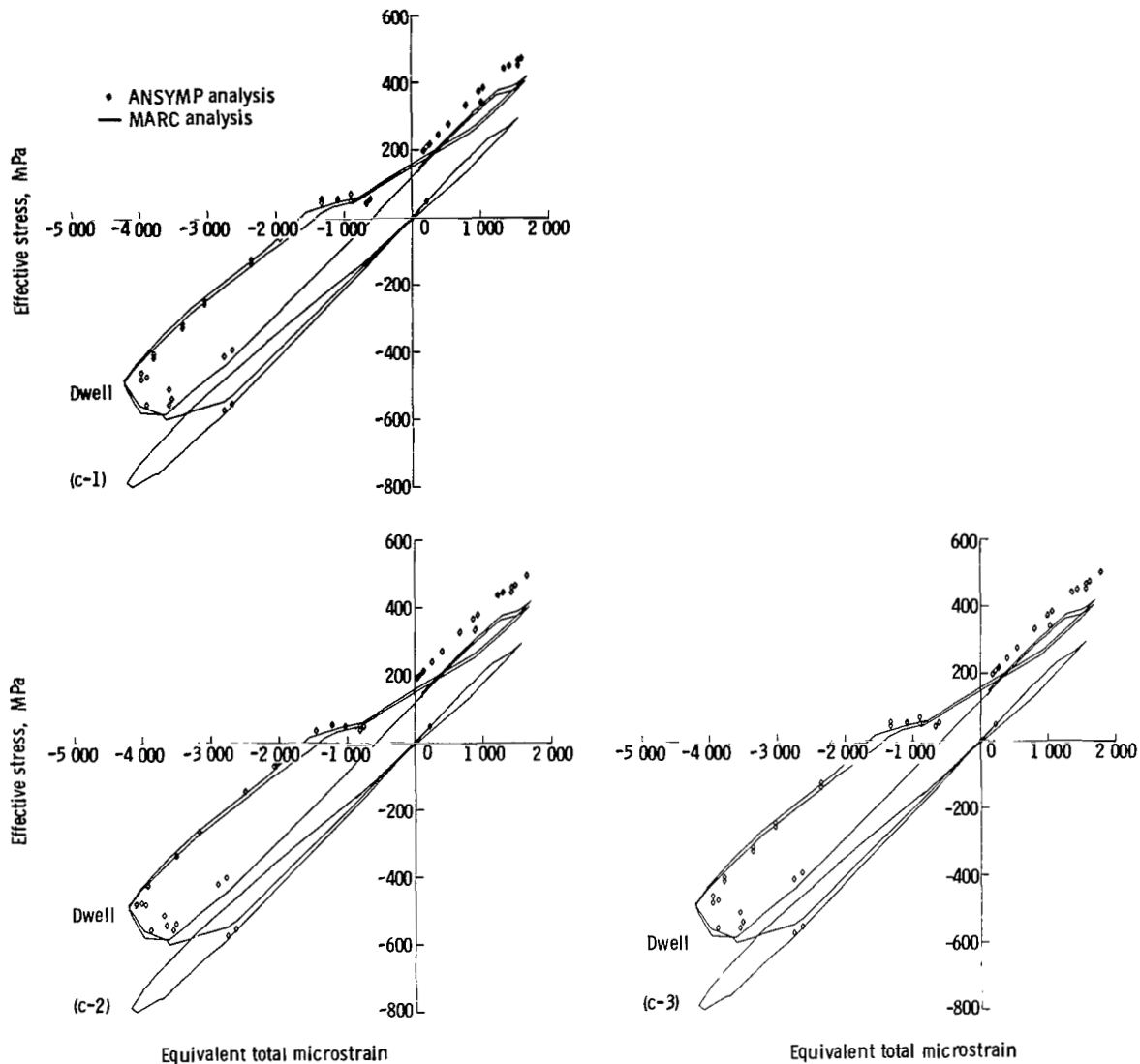


Figure 7. — Comparison of ANSYMP and MARC analysis results for wedge problem.



(c-1) Creep option (1).  
(c-2) Creep option (2).  
(c-3) Creep option (3).  
(c) Dwell time at minimum strain.

Figure 7. — Concluded.

extreme sensitivity of creep computations to small variations in stress. The maximum tensile stresses predicted from ANSYMP for the elastic-plastic case (fig. 7(a)) were in an elastic region of the cycle and were not accurate enough to use for creep calculations. Better agreement between ANSYMP and MARC elastic-plastic-creep solutions is shown in figure 7(c) for dwell times applied at the minimum strain level of the cycle. This is due to the better agreement between ANSYMP and MARC stress predictions in compressive yield shown in figure 7(a) because stresses in the plastic region are less sensitive to errors in strain than in the elastic region. All

three creep options gave reasonably good results for this case.

## Summary of Results

A simplified analysis procedure was developed for calculating the stress-strain history at the critical location of a thermomechanically cycled structure. A FORTRAN IV computer program, ANSYMP, was created to implement this procedure. The general conclusions and observations that were drawn from the evaluation of the method are as follows:

1. The predicted stress-strain response showed good to excellent agreement with elastic-plastic finite-element analysis solutions using the MARC program.

2. The predicted creep response showed generally good agreement with comparable MARC analytical results. However, the accuracy of the creep calculations was very sensitive to variations in the calculated effective stresses from the MARC solution for the elastic-plastic case without creep. The creep option averaging the effects of stress relaxation at constant stress and cumulative creep at constant total strain demonstrated the most consistent agreement with MARC creep computations except for strain-controlled problems.

3. Mean cyclic stress predictions were in considerably better agreement with MARC nonlinear analysis results than mean stresses obtained from elastic solutions.

4. Nonlinear stress-strain histories were computed from the ANSYMP program with less than 1 percent of the CPU time required by the MARC program.

Lewis Research Center  
National Aeronautics and Space Administration  
Cleveland, Ohio, October 31, 1983

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TABLE I. – SAMPLE PROGRAM INPUT

[Number of increments of elastic input data, NN, 66; number of increments with dwell times, NCRP, 2; number of subincrements with dwell times subdivided for creep calculations, ICRP, 10; selected creep option, ICREEP, 3; pointer that refers to the type of problem to be solved and the set of material properties to be used in the analysis, KKK, 4.]

INC	TEMP F	STRESS PSI	TOTAL STRAIN				
1	651.	7119.	2.147E-4	45	1900.	-47070.	-1.939E-3
2	1170.	-79740.	-2.636E-3	46	1940.	-45290.	-1.889E-3
3	1230.	-82590.	-2.762E-3	47	1950.	-31770.	-1.332E-3
4	1380.	-102400.	-3.521E-3	48	1960.	-25930.	-1.091E-3
5	1420.	-102500.	-3.563E-3	49	1970.	-21390.	-9.038E-4
6	1470.	-108200.	-3.892E-3	50	1970.	-14730.	-6.224E-4
7	1630.	-106400.	-3.971E-3	51	1920.	-16350.	-6.767E-4
8	1720.	-98280.	-3.788E-3	52	1700.	26570.	1.017E-3
9	1790.	-85360.	-3.367E-3	53	1560.	42760.	1.556E-3
10	1850.	-75300.	-3.038E-3	54	1490.	49990.	1.778E-3
11	1870.	-57920.	-2.359E-3	55	1460.	45770.	1.611E-3
12	1900.	-47070.	-1.939E-3	56	1420.	44800.	1.559E-3
13	1940.	-45290.	-1.889E-3	57	1260.	42440.	1.426E-3
14	1950.	-31770.	-1.332E-3	58	1120.	41240.	1.349E-3
15	1960.	-25930.	-1.091E-3	59	1010.	32770.	1.051E-3
16	1970.	-21390.	-9.038E-4	60	930.	31180.	9.848E-4
17	1970.	-14730.	-6.224E-4	61	866.	25480.	7.954E-4
18	1920.	-16350.	-6.767E-4	62	826.	17220.	5.336E-4
19	1700.	26570.	1.017E-3	63	770.	12640.	3.882E-4
20	1560.	42760.	1.556E-3	64	737.	8687.	2.654E-4
21	1490.	49990.	1.778E-3	65	690.	8504.	2.580E-4
22	1460.	45770.	1.611E-3	66	666.	5699.	1.723E-4
23	1420.	44800.	1.559E-3				
24	1260.	42440.	1.426E-3	JL	TIMINC		
25	1120.	41240.	1.349E-3	7	120.		
26	1010.	32770.	1.051E-3	40	120.		
27	930.	31180.	9.848E-4				
28	866.	25480.	7.954E-4				
29	826.	17220.	5.336E-4				
30	770.	12640.	3.882E-4				
31	737.	8687.	2.654E-4				
32	690.	8504.	2.580E-4				
33	666.	5699.	1.723E-4				
34	651.	7119.	2.147E-4				
35	1170.	-79740.	-2.636E-3				
36	1230.	-82590.	-2.762E-3				
37	1380.	-102400.	-3.521E-3				
38	1420.	-102500.	-3.563E-3				
39	1470.	-108200.	-3.892E-3				
40	1630.	-106400.	-3.971E-3				
41	1720.	-98280.	-3.788E-3				
42	1790.	-85360.	-3.367E-3				
43	1850.	-75300.	-3.038E-3				
44	1870.	-57920.	-2.359E-3				

TABLE II. - SAMPLE PROGRAM OUTPUT

NN= 66    NCRP= 2    ICRP= 10    ICREEP= 3    KKK= 4  
 INCREMENT NUMBER= 7    DWELL TIME= 120.00000  
 INCREMENT NUMBER= 40    DWELL TIME= 120.00000

INC	TEMP F	STRESS PSI	TOTAL STRAIN	PLASTIC STRAIN	CREEP STRAIN
1	651.	7119.	0.215E-03	0.000E+00	0.000E+00
2	1170.	-79740.	-0.264E-02	0.000E+00	0.000E+00
3	1230.	-82590.	-0.276E-02	0.000E+00	0.000E+00
4	1380.	-77372.	-0.352E-02	0.000E+00	0.000E+00
5	1420.	-79931.	-0.356E-02	-0.354E-03	0.000E+00
6	1470.	-79999.	-0.389E-02	-0.629E-03	0.000E+00
7	1630.	-69287.	-0.397E-02	-0.985E-03	0.000E+00
8	1630.	-67793.	-0.404E-02	-0.986E-03	-0.135E-03
9	1720.	-59673.	-0.386E-02	-0.986E-03	-0.135E-03
10	1790.	-46753.	-0.343E-02	-0.986E-03	-0.135E-03
11	1850.	-36693.	-0.311E-02	-0.986E-03	-0.135E-03
12	1870.	-19313.	-0.243E-02	-0.986E-03	-0.135E-03
13	1900.	-8463.	-0.201E-02	-0.986E-03	-0.135E-03
14	1940.	-6683.	-0.196E-02	-0.986E-03	-0.135E-03
15	1950.	6837.	-0.140E-02	-0.986E-03	-0.135E-03
16	1960.	7699.	-0.116E-02	-0.986E-03	-0.135E-03
17	1970.	11498.	-0.972E-03	-0.139E-02	-0.135E-03
18	1970.	8038.	-0.690E-03	-0.948E-03	-0.135E-03
19	1920.	6418.	-0.744E-03	-0.948E-03	-0.135E-03
20	1700.	49338.	0.949E-03	-0.948E-03	-0.135E-03
21	1560.	65528.	0.149E-02	-0.948E-03	-0.135E-03
22	1490.	72758.	0.171E-02	-0.948E-03	-0.135E-03
23	1460.	68538.	0.154E-02	-0.948E-03	-0.135E-03
24	1420.	67568.	0.149E-02	-0.948E-03	-0.135E-03
25	1260.	65208.	0.136E-02	-0.948E-03	-0.135E-03
26	1120.	64008.	0.128E-02	-0.948E-03	-0.135E-03
27	1010.	55538.	0.983E-03	-0.948E-03	-0.135E-03
28	930.	53948.	0.917E-03	-0.948E-03	-0.135E-03
29	866.	48248.	0.728E-03	-0.948E-03	-0.135E-03
30	826.	39988.	0.466E-03	-0.948E-03	-0.135E-03
31	770.	35408.	0.320E-03	-0.948E-03	-0.135E-03
32	737.	31455.	0.198E-03	-0.948E-03	-0.135E-03
33	690.	31272.	0.190E-03	-0.948E-03	-0.135E-03
34	666.	28467.	0.105E-03	-0.948E-03	-0.135E-03
35	651.	29887.	0.147E-03	-0.948E-03	-0.135E-03
36	1170.	-56972.	-0.270E-02	-0.948E-03	-0.135E-03
37	1230.	-59822.	-0.283E-02	-0.948E-03	-0.135E-03
38	1380.	-77474.	-0.359E-02	-0.948E-03	-0.135E-03
39	1420.	-73278.	-0.363E-02	-0.948E-03	-0.135E-03
40	1470.	-68070.	-0.396E-02	-0.948E-03	-0.135E-03
41	1630.	-68300.	-0.404E-02	-0.963E-03	-0.135E-03
42	1630.	-68311.	-0.404E-02	-0.963E-03	-0.137E-03
43	1720.	-60191.	-0.386E-02	-0.963E-03	-0.137E-03
44	1790.	-47271.	-0.344E-02	-0.963E-03	-0.137E-03

TABLE II. – Concluded.

45	1850.	-37211.	-0.311E-02	-0.963E-03	-0.137E-03
46	1870.	-19831.	-0.243E-02	-0.963E-03	-0.137E-03
47	1900.	-8981.	-0.201E-02	-0.963E-03	-0.137E-03
48	1940.	-7201.	-0.196E-02	-0.963E-03	-0.137E-03
49	1950.	6319.	-0.140E-02	-0.963E-03	-0.137E-03
50	1960.	8577.	-0.116E-02	-0.963E-03	-0.137E-03
51	1970.	8000.	-0.972E-03	-0.963E-03	-0.137E-03
52	1970.	8090.	-0.691E-03	-0.951E-03	-0.137E-03
53	1920.	6470.	-0.745E-03	-0.951E-03	-0.137E-03
54	1700.	49390.	0.948E-03	-0.951E-03	-0.137E-03
55	1560.	65580.	0.149E-02	-0.951E-03	-0.137E-03
56	1490.	72810.	0.171E-02	-0.951E-03	-0.137E-03
57	1460.	68590.	0.154E-02	-0.951E-03	-0.137E-03
58	1420.	67620.	0.149E-02	-0.951E-03	-0.137E-03
59	1260.	65260.	0.136E-02	-0.951E-03	-0.137E-03
60	1120.	64060.	0.128E-02	-0.951E-03	-0.137E-03
61	1010.	55590.	0.982E-03	-0.951E-03	-0.137E-03
62	930.	54000.	0.916E-03	-0.951E-03	-0.137E-03
63	866.	48300.	0.727E-03	-0.951E-03	-0.137E-03
64	826.	40040.	0.465E-03	-0.951E-03	-0.137E-03
65	770.	35460.	0.320E-03	-0.951E-03	-0.137E-03
66	737.	31507.	0.197E-03	-0.951E-03	-0.137E-03
67	690.	31324.	0.189E-03	-0.951E-03	-0.137E-03
68	666.	28519.	0.104E-03	-0.951E-03	-0.137E-03

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16. Abstract  Development was extended of a simplified inelastic analysis computer program (ANSYMP) for predicting the stress-strain history at the critical location of a thermomechanically cycled structure from an elastic solution. The program uses an iterative and incremental procedure to estimate the plastic strains from the material stress-strain properties and a plasticity hardening model. Creep effects can be calculated on the basis of stress relaxation at constant strain, creep at constant stress, or a combination of stress relaxation and creep accumulation. The simplified method was exercised on a number of problems involving uniaxial and multiaxial loading, isothermal and nonisothermal conditions, dwell times at various points in the cycles, different materials, and kinematic hardening. Good agreement was found between these analytical results and nonlinear finite-element solutions for these problems. The simplified analysis program used less than 1 percent of the CPU time required for a nonlinear finite-element analysis.					
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